Electrical Heating with Horizontal Wells
The Heat Transfer Problem

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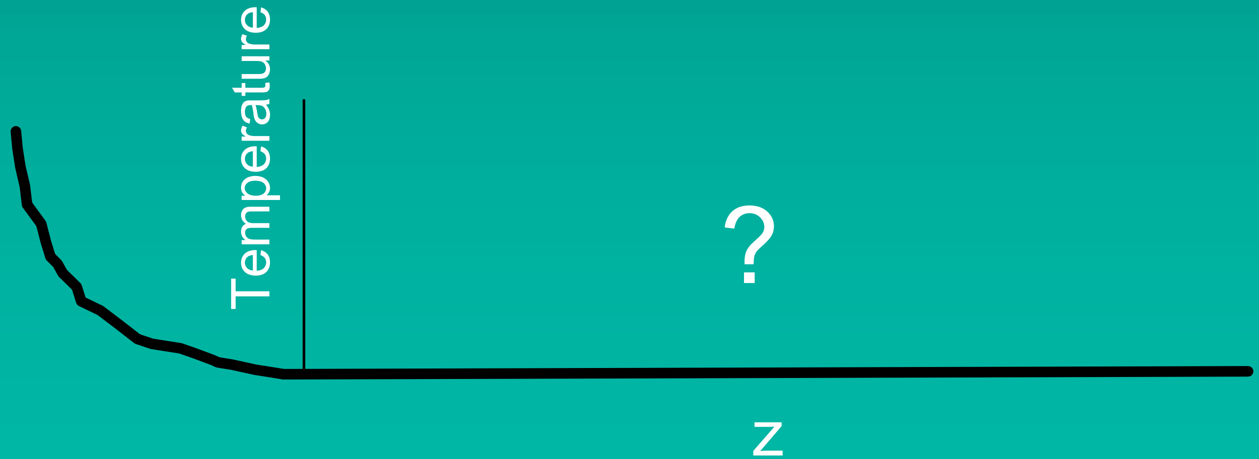
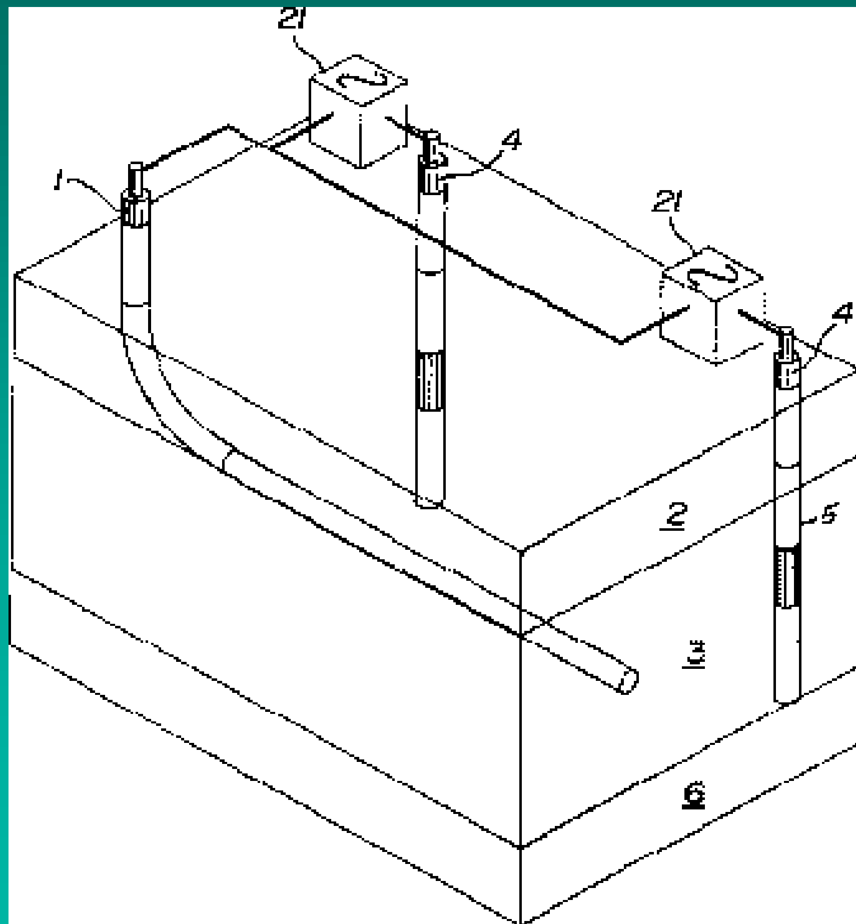
Outline

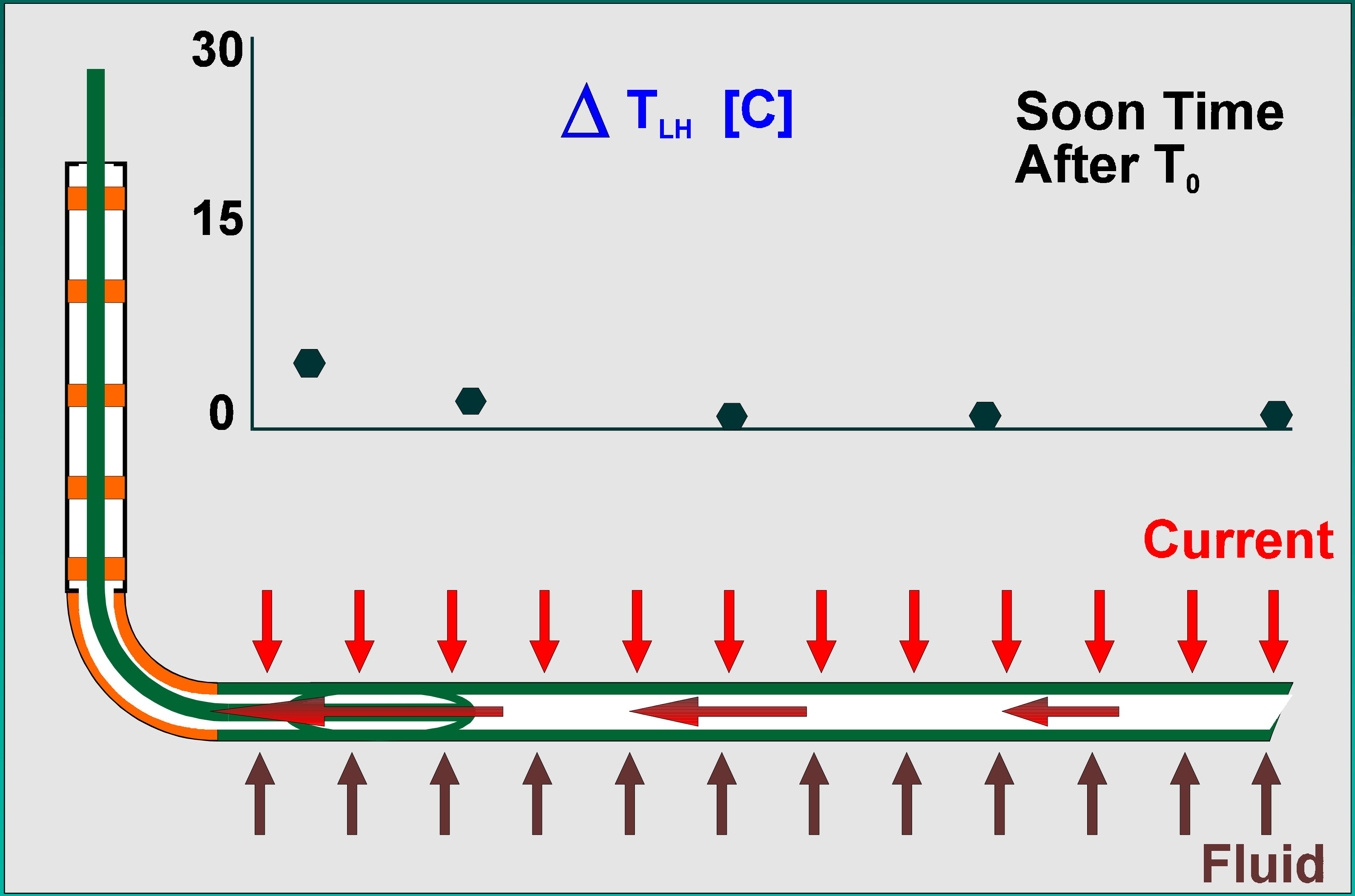
- Present our mathematical solution of the axial heat transfer problem for electrical heating of a horizontal well.
- Present some results.



TEXCAN Horizontal Well Electrical Heating Technology



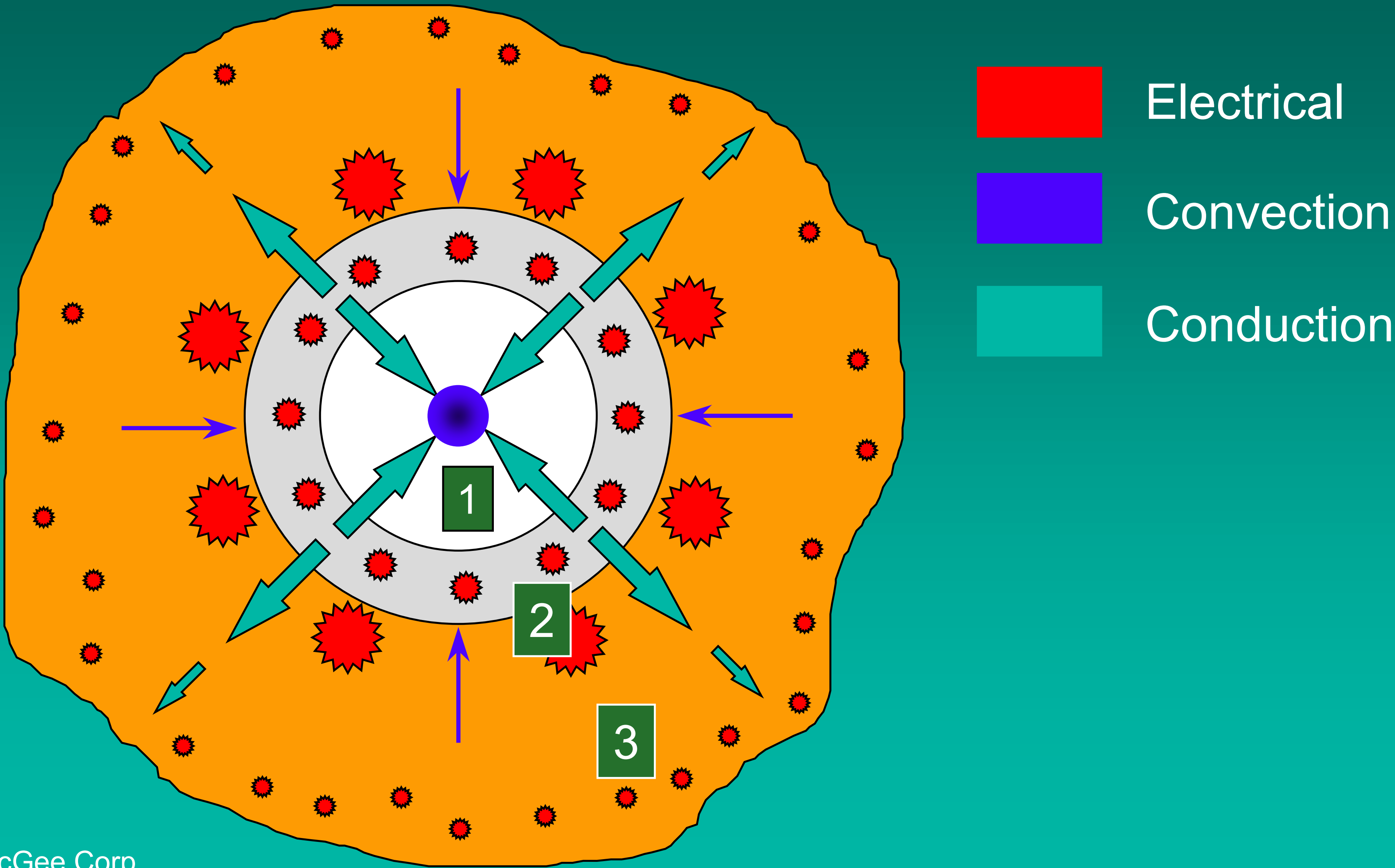
Process Dynamics at T_0 +

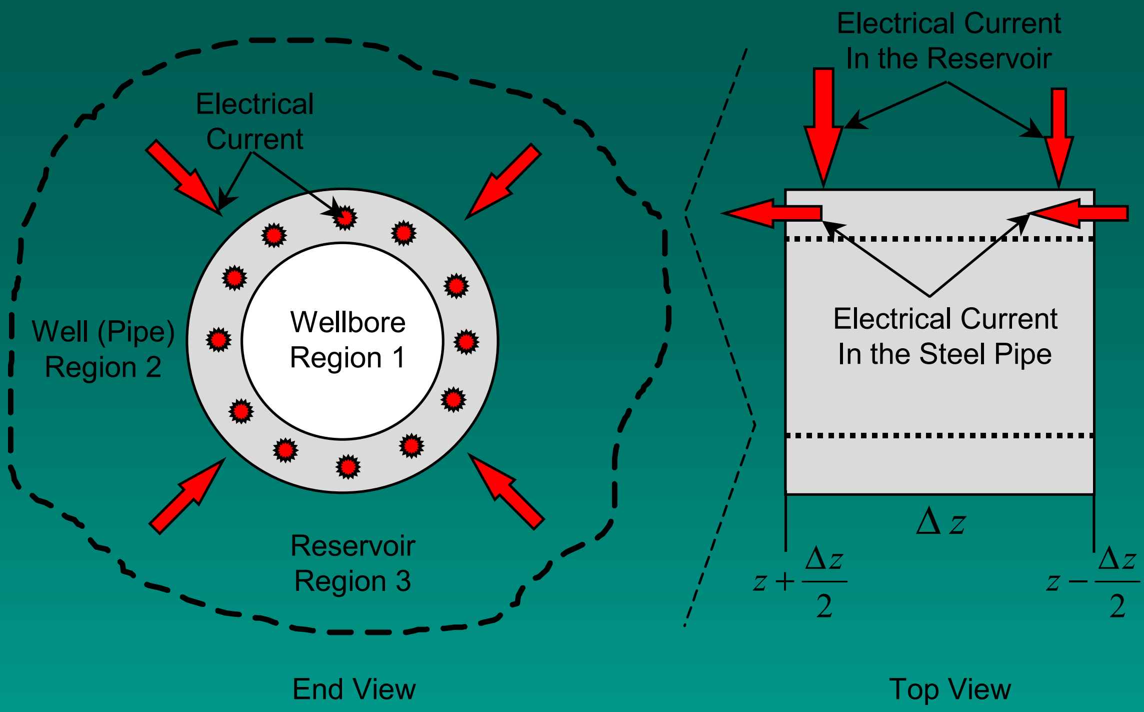


Calculation Procedure

- Derive the Heat Transfer PDE for each region.
- Obtain the Laplace Transforms of these equations.
- Solve the above equations.
- Construct a system of linear equations and determine the unknown coefficients using the boundary conditions and matrix algebra.
- Determine the energy transfer into wellbore and solve the transformed heat transfer equation there.
- Invert the Laplace solution into the time domain.

Heat Transfer Mechanisms

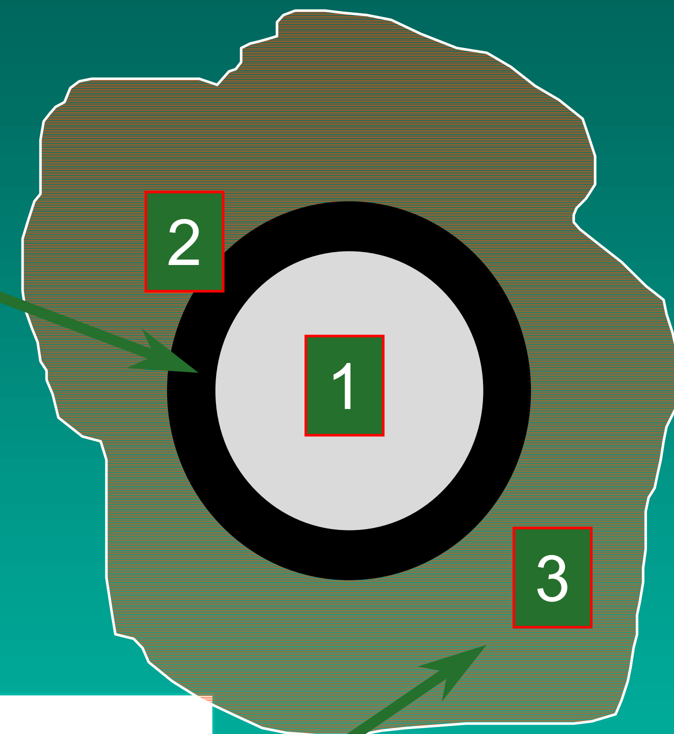




Electrical Current
Well (Pipe) Region 2
Wellbore Region 1
Reservoir Region 3
End View
Electrical Current In the Reservoir
Electrical Current In the Steel Pipe
Δz
$z + \frac{\Delta z}{2}$
$z - \frac{\Delta z}{2}$
Top View

Heat Transfer Equations
Regions 2 and 3

$$\lambda_s \frac{\partial^2 T_2}{\partial r^2} + \frac{\lambda_s}{r}\frac{\partial T_2}{\partial r} + \dot{q}_2(z) = \rho C_s \frac{\partial T_2}{\partial t}$$

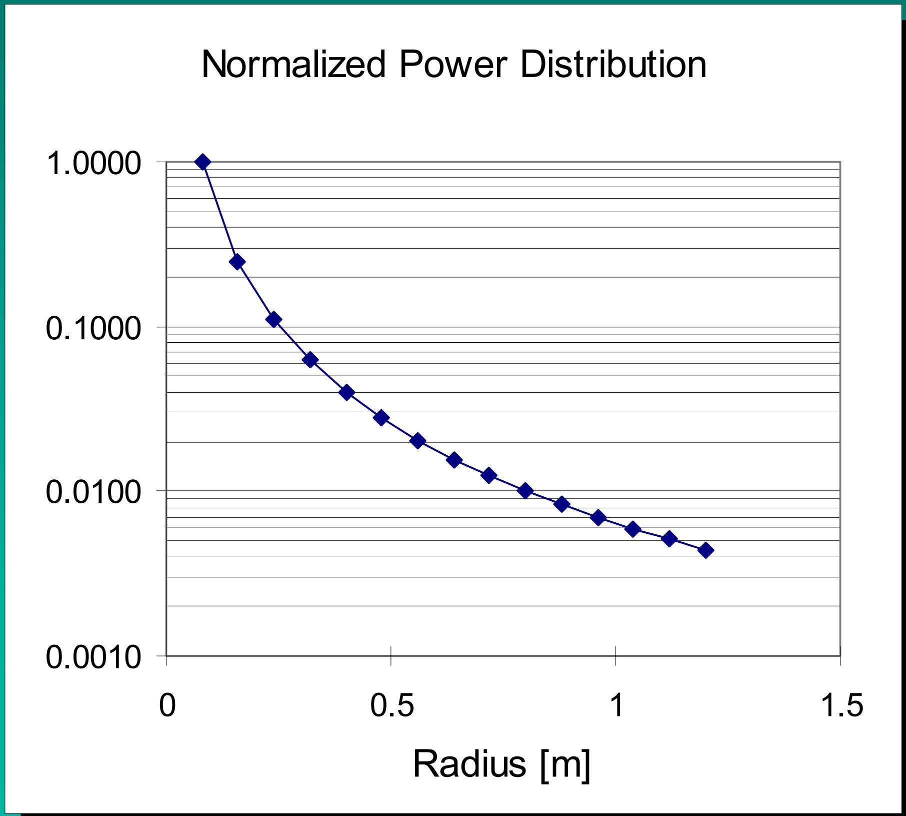


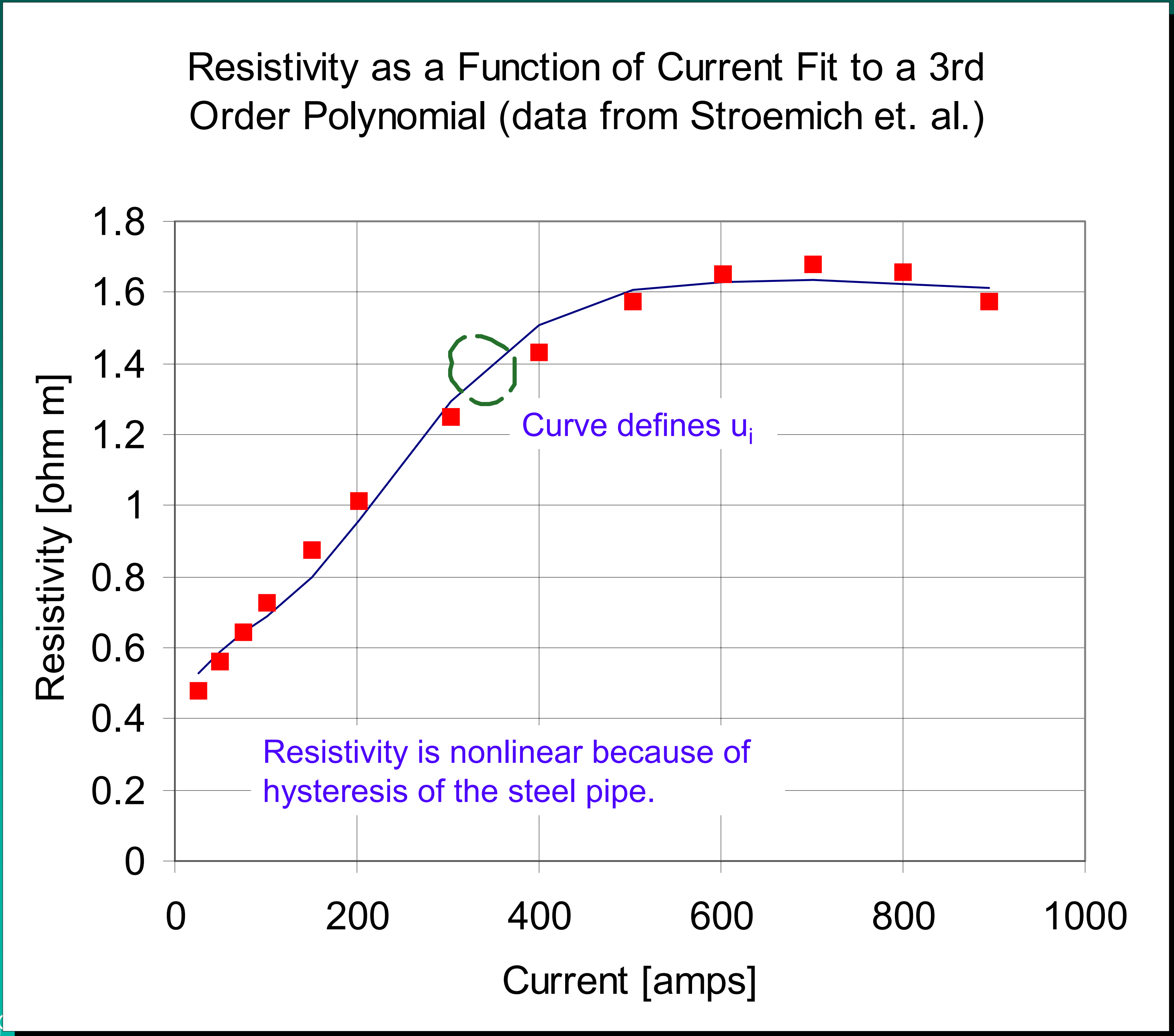
$$\lambda_r \frac{\partial^2 T_3}{\partial r^2} + \left(\lambda_r + \rho C_r \frac{q_f}{2\pi l}\right)\frac{1}{r}\frac{\partial T_3}{\partial r} + \dot{q}_3(r) = \rho C_r \frac{\partial T_3}{\partial t}$$

The Energy Source In Region 3

$$\dot{q}_2(z) = \rho_r \left|J(r)\right|^2 = \rho_r \left(\frac{I_t}{2\pi r l}\right)^2$$

At a distance of 10 wellbore radii from the wellbore, the heating rate is reduced by a factor of 100!





Resistivity as a Function of Current Fit to a 3rd Order Polynomial (data from Stroemich et. al.)
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Resistivity [ohm m]
0
200
400
600
800
1000
Current [amps]
Curve defines u_i
Resistivity is nonlinear because of hysteresis of the steel pipe.

The Energy Source In Region 2

axial accumulation of current

$$\dot{q}_2(z) = \rho_s\big(I(z)\big)\cdot\frac{I(z)^2}{A_s^2} = \rho_s(z)\cdot\frac{I_t^2}{A_s^2}\left(\frac{z}{l}\right)^2$$

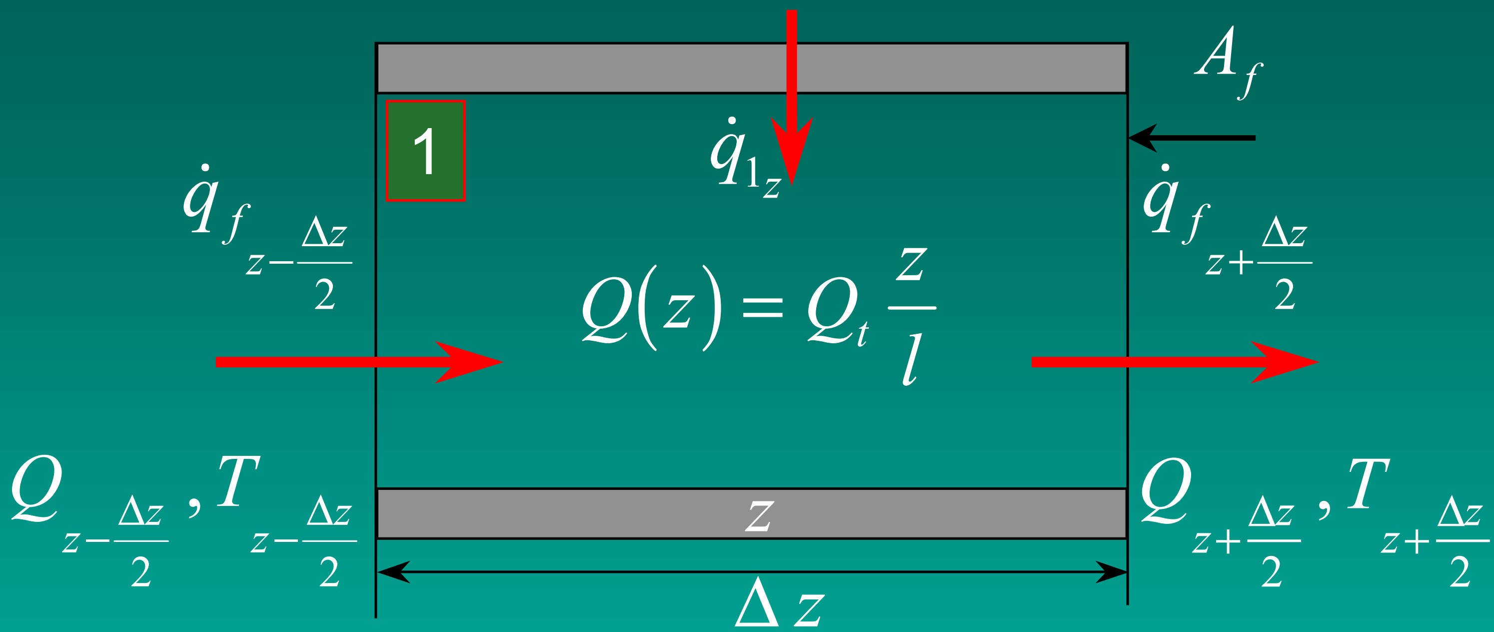
nonlinear hysteresis effects

$$\rho_s\big(I(z)\big) = \rho_s(z) = \sum_{i=0}^{n} u_i\left(\frac{I_t}{l}\right)^i z^i$$

$$I_t\left(\frac{z}{l}\right)$$

$$\dot{q}_2(z) = \frac{1}{A_s^2}\sum_{i=0}^{n} u_i\left(\frac{I_t}{l}\right)^{i+2} z^{i+2}$$

Heat Transfer Equation - Region 1

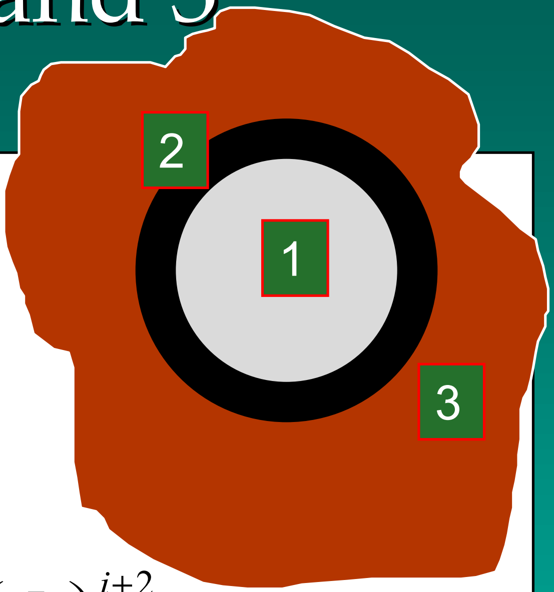


$$z \frac{\partial T_1(z)}{\partial z} = -\alpha \frac{\partial T_1(z)}{\partial t} + 2\alpha \frac{\lambda_s}{\rho C_f} \frac{1}{r_i} \frac{\partial T_2(r)}{\partial r}\bigg|_{r=r_i}$$

$$\alpha = \frac{A_f \ell}{Q_t}$$

$\dot{q}_{1_z}$

Transformed Heat Transfer Equations Regions 1, 2, and 3



1

$$z\frac{d\hat{T}_1(z)}{dz}+s\alpha\hat{T}_1=\alpha T_0+2\alpha\frac{\lambda_s}{\rho C_f}\frac{1}{r_i}\frac{d\hat{T}_2(r)}{dr}\Big|_{r=r_i}$$

2

$$\lambda_s\frac{d^2\hat{T}_2}{dr^2}+\frac{\lambda_s}{r}\frac{d\hat{T}_2}{dr}-s\rho C_s\hat{T}_2=-\rho C_s T_0-\frac{1}{sA_s^2}\sum_{i=0}^{n}u_i\left(\frac{I_t}{l}\right)^{i+2}z^{i+2}$$

3

$$\lambda_r\frac{d^2\hat{T}_3}{dr^2}+\left(\lambda_r+\rho C_r\frac{q_f}{2\pi l}\right)\frac{1}{r}\frac{d\hat{T}_3}{dr}-s\rho C_r\hat{T}_3=-\rho C_r T_0-\frac{\rho_r}{s}\left(\frac{I_t}{2\pi l}\right)^2\frac{1}{r^2}$$

Solution of Transformed Heat Transfer Equations in Regions 1 and 2

2

$$\hat{T}_2(r,z,s) = C(s)\,\mathrm{I}_0\left(r\sqrt{s\frac{\rho C_s}{\lambda_s}}\right) + D(s)\,\mathrm{K}_0\left(r\sqrt{s\frac{\rho C_s}{\lambda_s}}\right) + \frac{T_0}{s} + \sum_{i=0}^{n}\frac{u_i}{s^2\beta_s\lambda_s A_s^2}\left(\frac{I_t}{l}\right)^{i+2} z^{i+2}$$

3

$$\hat{T}_3(r,s) = E(s)\frac{\mathrm{I_p}\left(r\sqrt{s\beta_r}\right)}{r^p} + F(s)\frac{\mathrm{K_p}\left(r\sqrt{s\beta_r}\right)}{r^p} - \gamma\frac{\mathrm{I_p}\left(r\sqrt{s\beta_r}\right)}{r^p}\int_{r_w}^{r}\frac{\mathrm{K_p}\left(x\sqrt{s\beta_r}\right)}{x^{2p-1}}dx + \gamma\frac{\mathrm{K_p}\left(r\sqrt{s\beta_r}\right)}{r^p}\int_{r_w}^{r}\frac{\mathrm{I_p}\left(x\sqrt{s\beta_r}\right)}{x^{2p-1}}dx$$

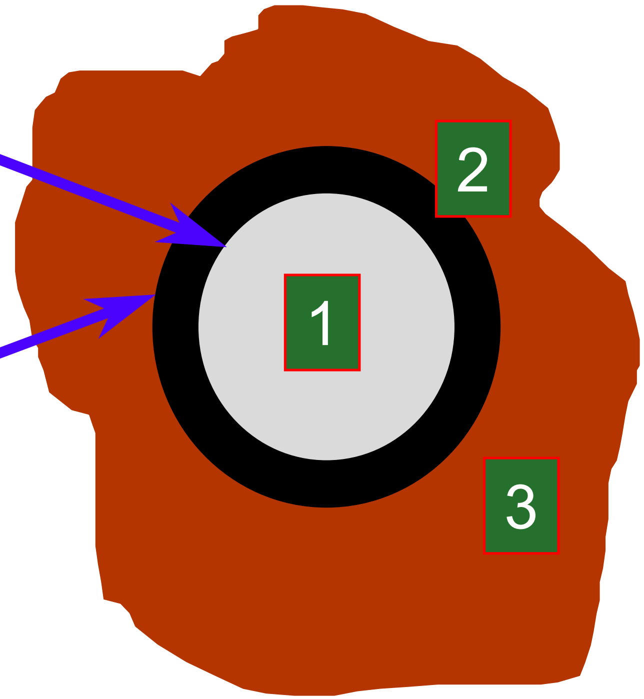
BC's and Expressions for C(s) and D(s)

$$\hat{T}_2(r_i, s) = \hat{T}_1(z, s)$$

Boundary Conditions

$$\hat{T}_2(r_w, s) = \hat{T}_3(r_w, s)$$

$$\lambda_s \frac{d\hat{T}_2(r,s)}{dr}\bigg|_{r=r_w} = \lambda_r \frac{d\hat{T}_3(r,s)}{dr}\bigg|_{r=r_w}$$



With C(s) and D(s) known, $T_2(r,s)$ is fully described.

$$C(s) = g_1(s)\left\{\hat{T}_1(z,s) - \frac{T_0}{s}\right\} + g_2(s)\sum_{i=0}^{n} \eta_i(s) z^{i+2}$$

$$D(s) = g_3(s)\left\{\hat{T}_1(z,s) - \frac{T_0}{s}\right\} + g_4(s)\sum_{i=0}^{n} \eta_i(s) z^{i+2}$$

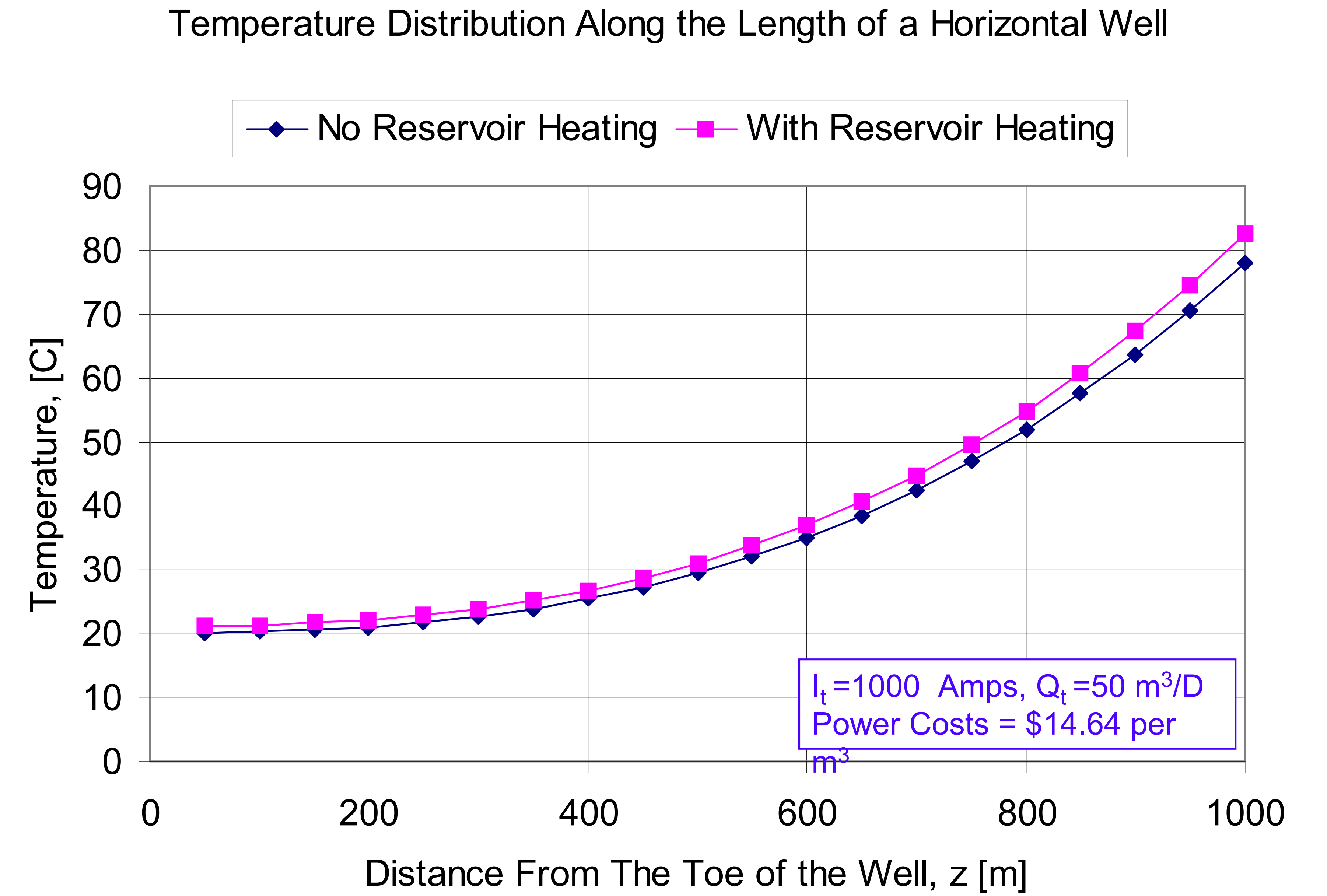
Explicit Expression for the Energy Source in Region 1 and Final Solution of the Transformed Equation

$$2\alpha \frac{\lambda_s}{\rho C_f} \frac{1}{r_i} \frac{d\hat{T}_2}{dr}\bigg|_{r=r_i} = \omega(s)\hat{T}_1(z,s) + \gamma(s)\sum_{i=0}^{n} \eta_i(s) z^{i+2} - \omega(s)\frac{T_0}{s}$$

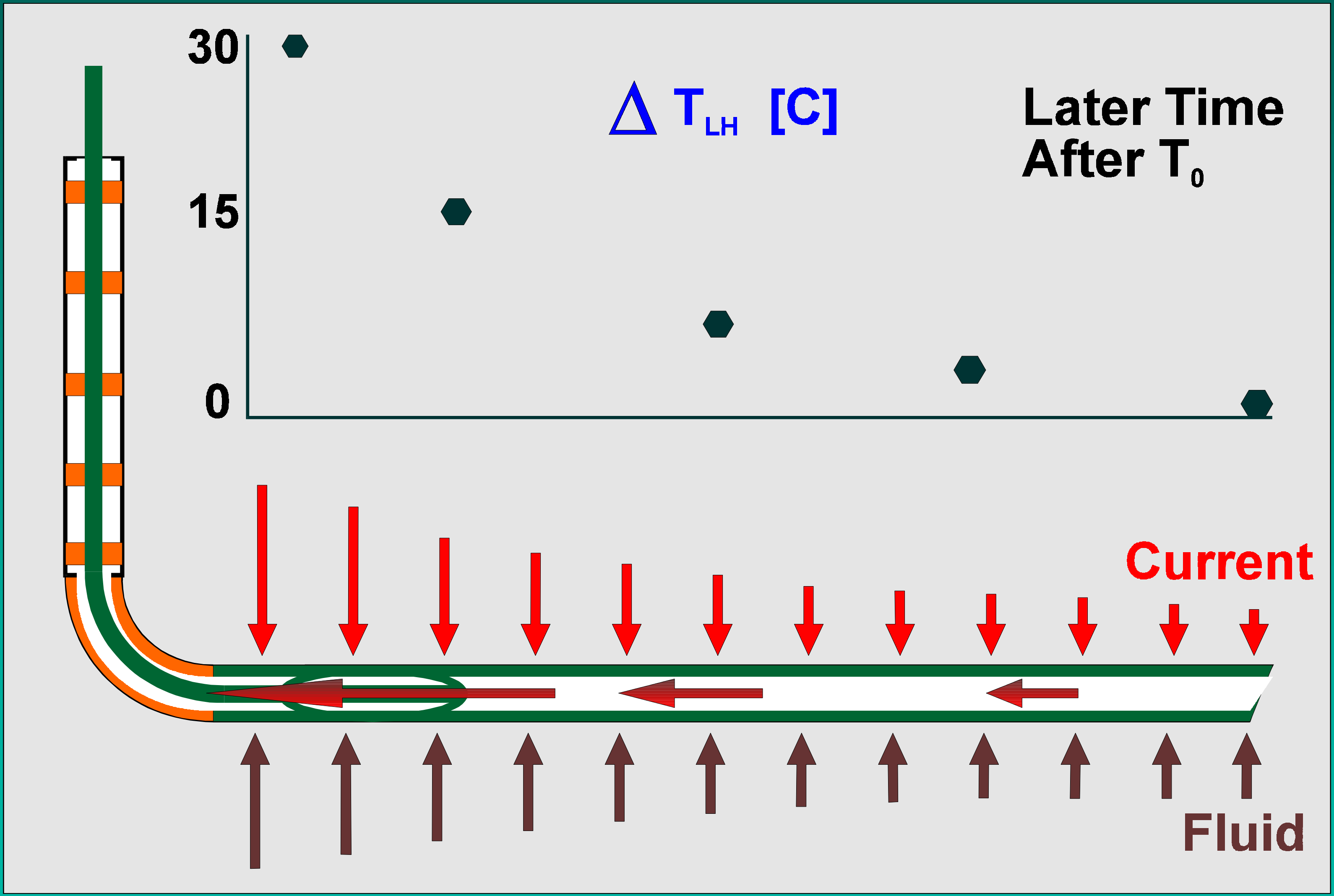
$$z\frac{d\hat{T}_1}{dz} + \left(s\alpha - \omega(s)\right)\hat{T}_1 = \frac{T_0}{s}\left(s\alpha - \omega(s)\right) + \gamma(s)\sum_{i=0}^{n} \eta_i(s) z^{i+2}$$

$$\hat{T}_1(z,s) = \frac{T_0}{s} + \gamma(s)\sum_{i=0}^{n} \frac{\eta_i(s) z^{i+2}}{i + s\alpha - \omega(s) + 2} -$$

$$\left(\frac{z}{z_0}\right)^{\omega(s)-s\alpha} \gamma(s)\sum_{i=0}^{n} \frac{\eta_i(s) z_0^{i+2}}{i + s\alpha - \omega(s) + 2}$$


Temperature Distribution Along the Length of a Horizontal Well
No Reservoir Heating
With Reservoir Heating
90
80
70
60
50
40
30
20
10
0
Temperature, [C]
0
200
400
600
800
1000
Distance From The Toe of the Well, z [m]
I_t =1000 Amps, Q_t =50 m^3/D
Power Costs = $14.64 per m^3

Process Dynamics at T_0 ++



Conclusions

- A semi-analytical model has been developed to determine the axial temperature distribution in a horizontal well that is being electrically heated.
- The temperature distribution is not uniform as presented, however the system can be modified for a more uniform temperature distribution
- The resulting temperature distribution, under normal operating conditions is determined by the current in the steel pipe as opposed to the current in the reservoir

Acknowledgments

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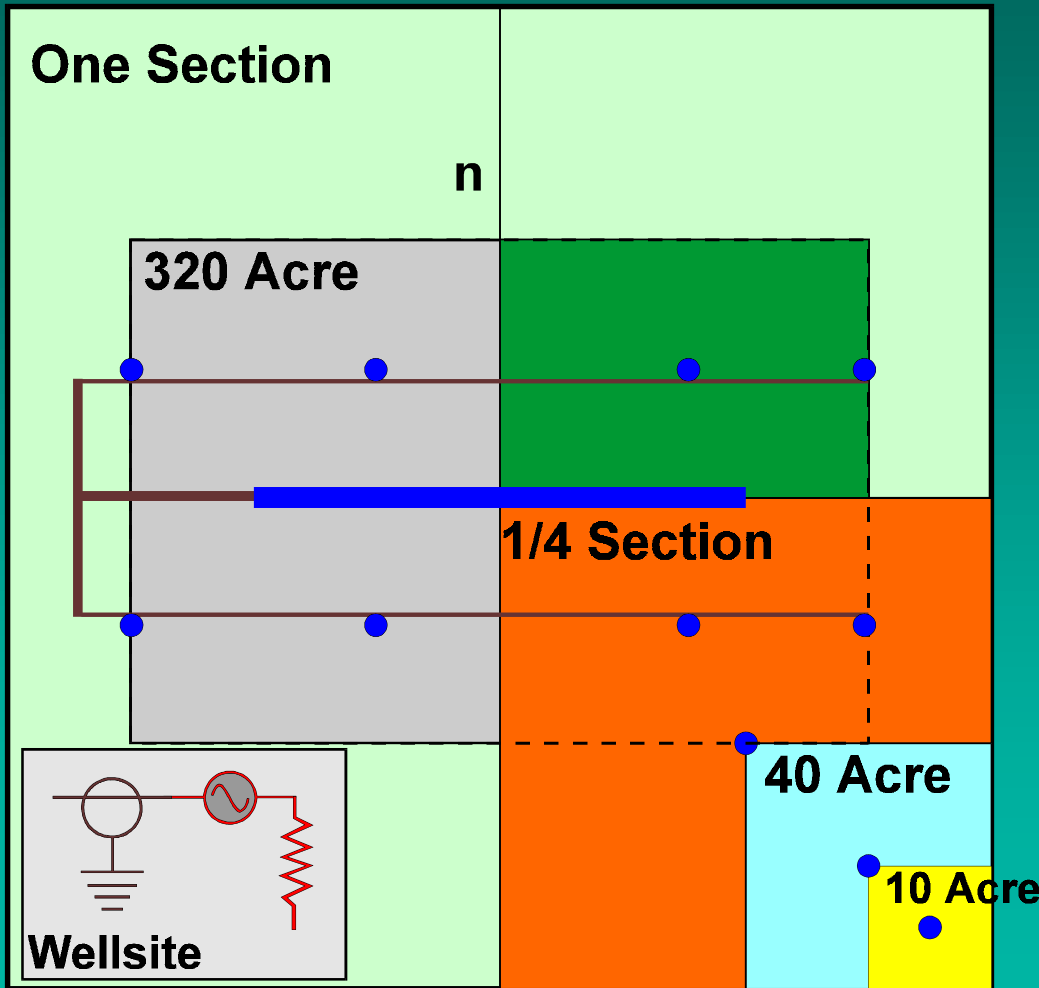
Commercial Implementation



Horizontal Well on 320 Acre Spacing

$$\frac{V}{H} = 8 : 1$$

Vertical Wells on 40 Acre Spacing

Ground Return